

Federated Learning: Incentives and Fairness

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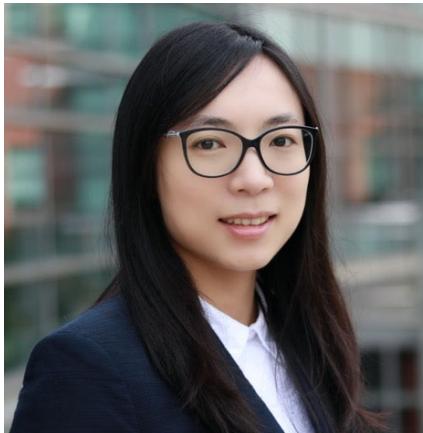
Zhuowen Yuan



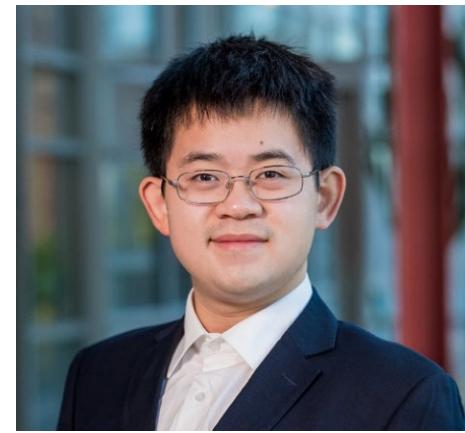
Jiaxin Song



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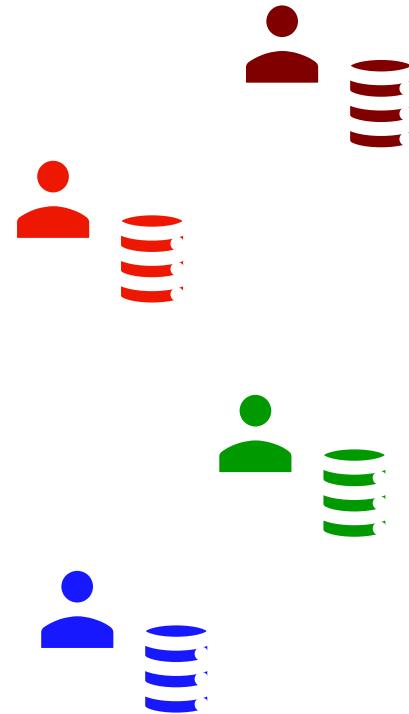
Bo Li



Linyi Li

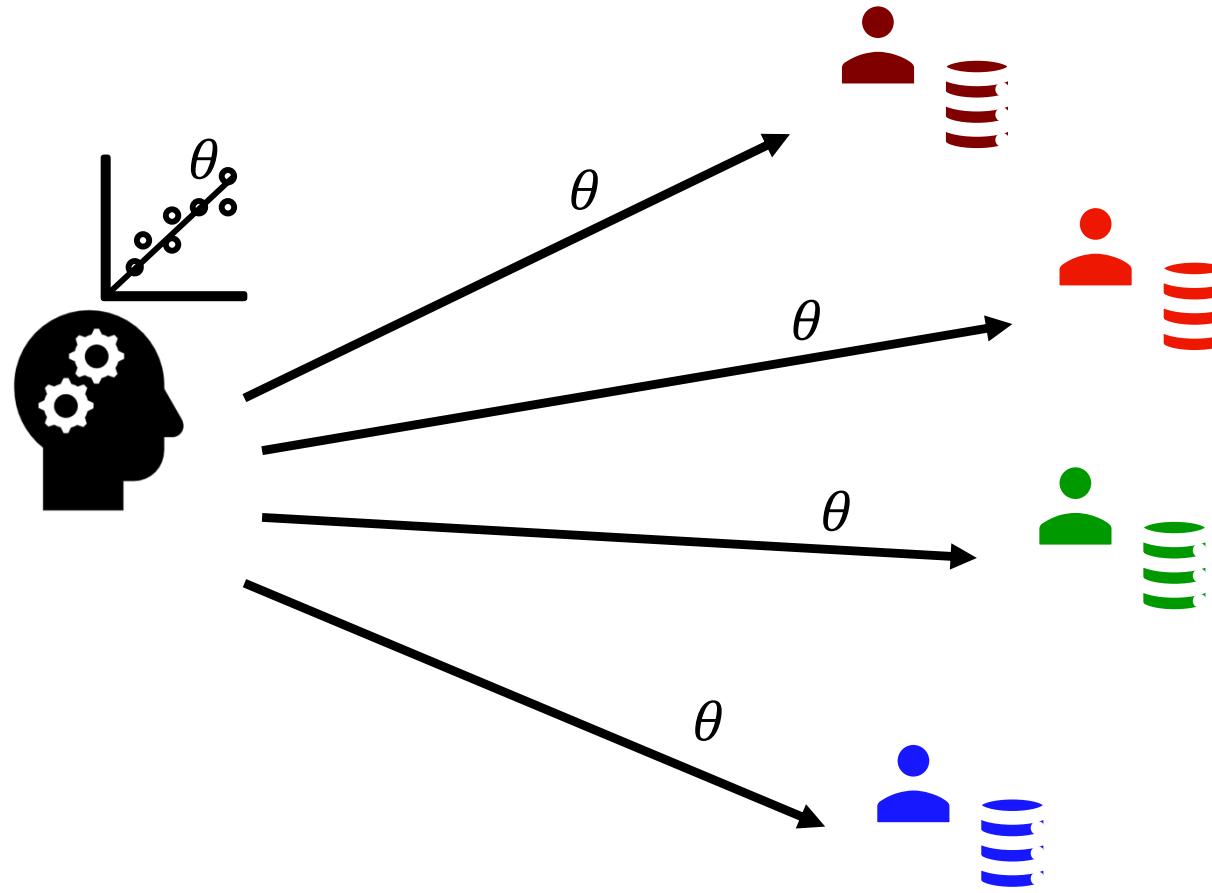


Ariel Procaccia



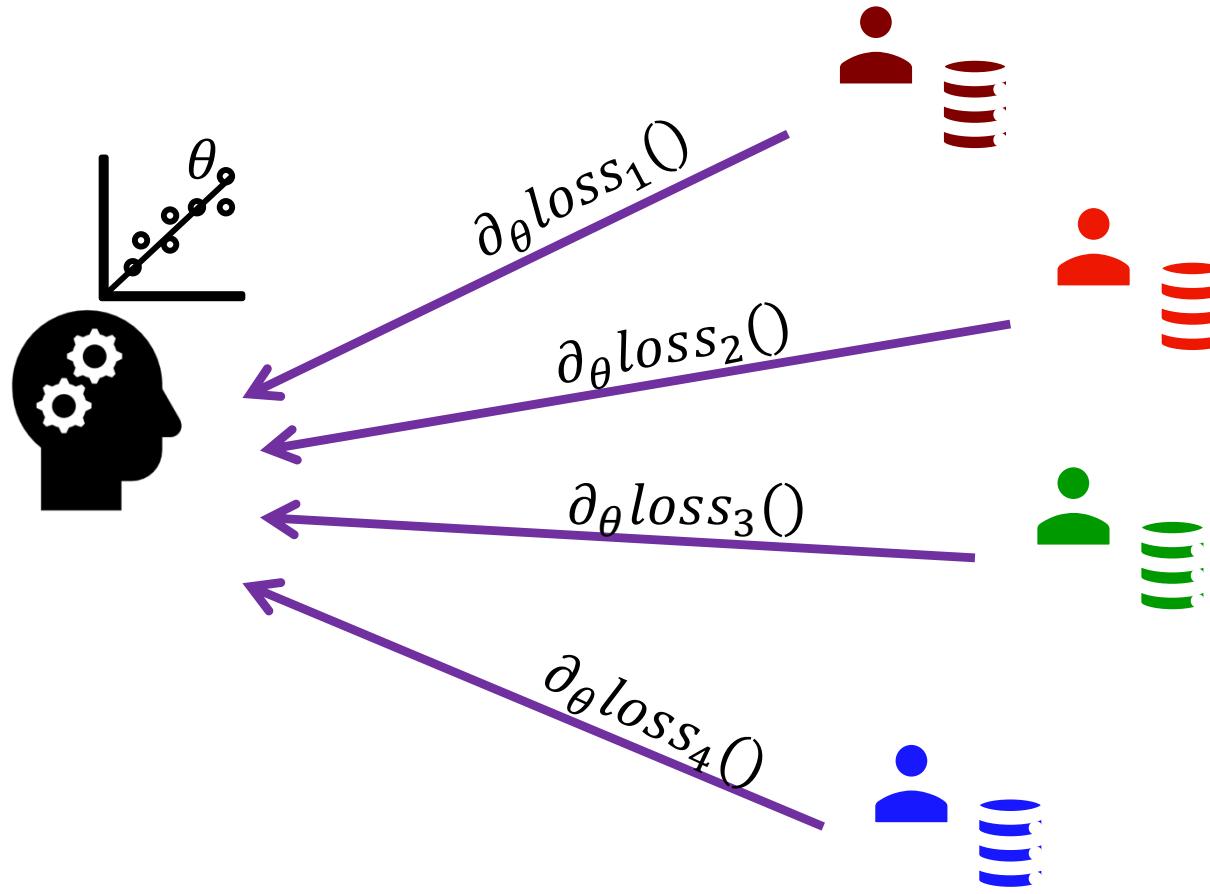
Goal: Learn using everyone's data without sharing it!

Federated Learning (FL)



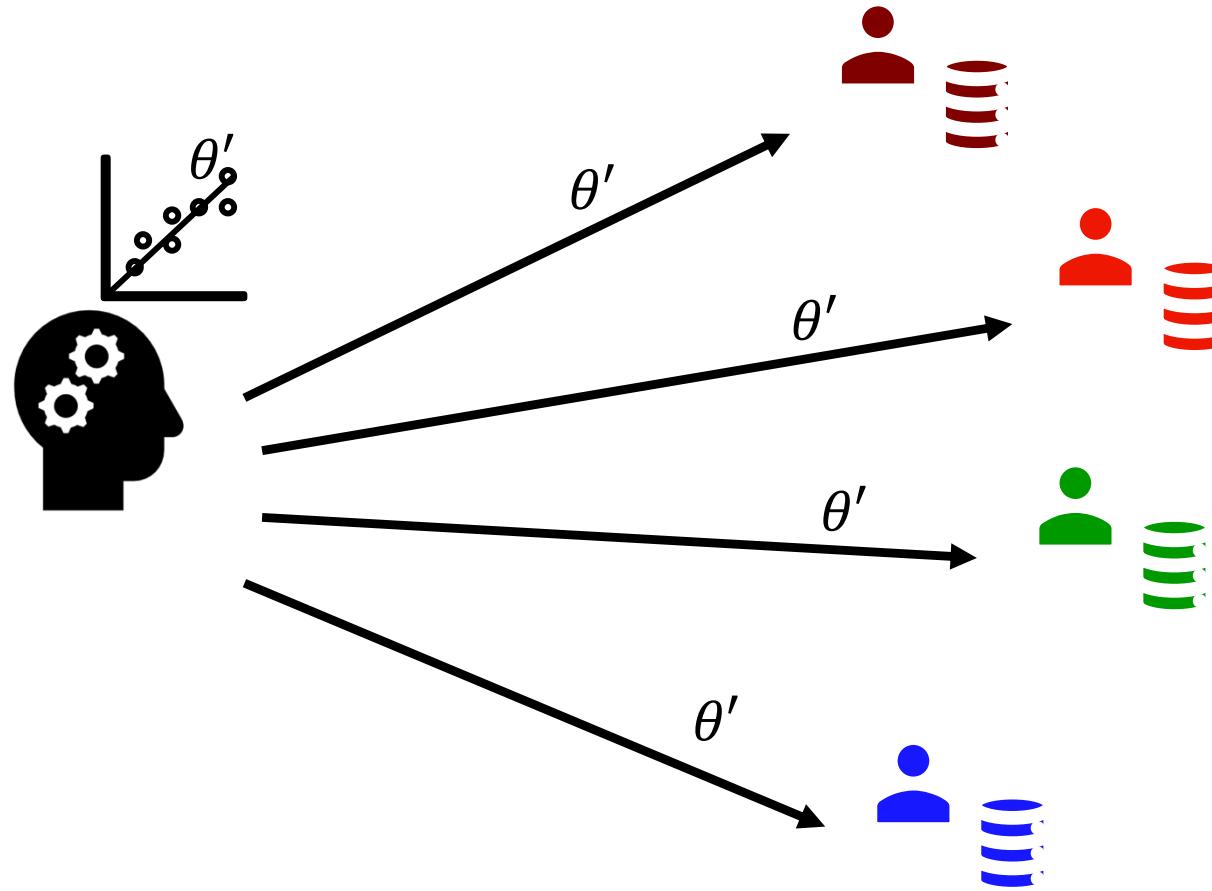
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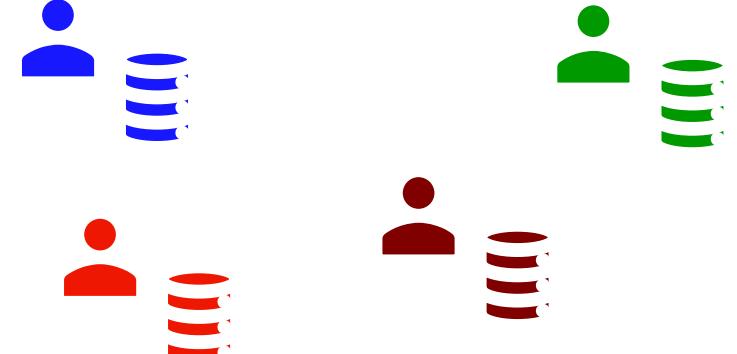


Goal: Learn using everyone's data without sharing it!



FL Model

- N : set of n clients
- P : set of models/classifiers
- Each agent $i \in A$ has
 - Dataset: D_i
 - Loss/error function: $loss_i: P \rightarrow R_+$
(On training data)



Goal: Learn $\theta \in P$ using the data from D_i s (indirectly)

FL History

Introduced by Google Deep Mind in 2016

- FedSGD:

$$\theta' = \theta - \eta \sum_i w_i \cdot \partial_{\theta} \text{loss}_i(.)$$

- FedAvg: Client updates the model and sends.
- **Extensive work:** Distributed, Privacy issues, Welfare, ...

Need not be fair/private/strategy-proof

Fair FL

[Donahue-Kleinberg'21]

Egalitarian Fairness:

$$\min_{\theta} \max_i \text{loss}_i(\theta)$$

Proportional/Equity-based Fairness:

$$n_i \text{loss}_i \approx n_j \text{loss}_j$$

(OR Equalize TPR/loss/...)

What if \exists noisy/adversarial agent with a lot of bad data?

[Du-Xu-Wu-Tong'21, Mohri-Sivesh-Suresh'19, Papadaki-Martine-Bertran-Shapiro'21, Xu-Lyu'20, Zafar-Valera-Gomez-Rodriguez-Gummadi'17, Zeng-Cheng-Lee'21, ...]

Fair FL

Egalitarian Fairness:

$$\min_{\theta} \max_i \text{loss}_i(\theta)$$

Proportional/Equity-based Fairness:

$$n_i \text{loss}_i \approx n_j \text{loss}_j$$

(OR Equalize TPR/loss/...)

Forced to
optimize for
the “bad”
agent!

May end up
harming
others.

What if \exists noisy/adversarial agent with a lot of bad data?

Fair FL \rightarrow Public Decision Making



- N : set of n **clients** \equiv **agents**
- P : set of **models/classifiers** \equiv **outcomes**
- Each agent $i \in A$ has
 - Dataset: D_i
 - Loss/error function** $loss_i \equiv$ **Utility function** $U_i = H - loss_i$
$$U_i: P \rightarrow R_+$$

Goal: Find $\theta \in P$ that is “liked” by all

CORE in Public Decisions

[Fain-Goel-Munagala'16, Fain-Munagala-Shah'18]

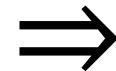
$$\theta^* \in P$$

- $S \subseteq N$ is a **Blocking Coalition** if $\exists \theta \in P$ s.t.

$$\forall i \in S, U_i(\theta) \geq U_i(\theta^*)$$

(with at least one strict inequality)

Distributed
Data



Data of agents in S is $\frac{|S|}{|N|}$ -representative
of the test data, and hence can only
ensure $\frac{|S|}{|N|}$ fraction of utility

CORE in Public Decisions

[Fain-Goel-Munagala'16, Fain-Munagala-Shah'18]

$$\theta^* \in P$$

- $S \subseteq N$ is a **Blocking Coalition** if $\exists \theta \in P$ s.t.

$$\forall i \in S, \frac{|S|}{|N|} U_i(\theta) \geq U_i(\theta^*)$$

(with atleast one strict inequality)

- **Outcome θ^* is in CORE if there is no blocking coalition.**

CORE in FL: Fair, Efficient, *Robust*

- $S \subseteq N$ is a **Blocking Coalition** if $\exists \theta \in P$ s.t.
 $\forall i \in S, \frac{|S|}{|N|} U_i(\theta) \geq U_i(\theta^*)$ with atleast one strict inequality
- θ^* is in **CORE** if there is no blocking coalition.

- **Pareto-Optimal (PO):** ($S = N$)
 $\nexists \theta \in P: \forall i \in N, U_i(\theta) \geq U_i(\theta^*)$ with atleast one inequality.
- **Pareto-Optimal (PO):** ($|S| = 1$)
 $\forall i \in N, U_i(\theta^*) \geq \frac{1}{n} \max_{\theta} U_i(\theta)$
- **Robust (to a few noisy/adversarial agents):**
 S = remaining good agents. S is non-blocking (happy)!

CORE in FL: Existence

[Chaudhury, Li, Kang, Li, M (NeurIPS'22)]

$$\phi(\theta) = \operatorname{argmax}_{c \in P} \sum_i \frac{U_i(c)}{U_i(\theta)}$$

Theorem 1. CORE exists if set $\phi(\theta)$ is a convex set $\forall \theta$.

Proof sketch.

1. Fixed points of ϕ are in CORE.
2. Apply Kakutani's fixed point to ϕ .

Covers: Concave U_i 's \equiv Convex $loss_i$'s
(Linear reg., Logistic reg., ...)

CORE in FL: Existence

[Chaudhury, Li, Kang, Li, M (NeurIPS'22)]

$$\phi(\theta) = \operatorname{argmax}_c \sum_i U_i(c)/U_i(\theta)$$

Claim. Fixed points of ϕ are in CORE.

Proof sketch. θ^* is FP $\Rightarrow \sum_i \frac{U_i(\theta)}{U_i(\theta^*)} \leq \sum_i \frac{U_i(\theta^*)}{U_i(\theta^*)} = n, \forall \theta$.

If $S \subseteq N$ *blocks* θ^* , then $\exists \theta \in P$ s.t.

$$\forall i \in S, \quad \frac{|S|}{n} U_i(\theta) \geq U_i(\theta^*) \Rightarrow \frac{U_i(\theta)}{U_i(\theta^*)} \geq \frac{n}{|S|}$$

(at least one strict)

$$\Rightarrow \sum_{i \in S} \frac{U_i(\theta)}{U_i(\theta^*)} > n !$$

CORE in FL: Computation

[Chaudhury, Li, Kang, Li, M (NeurIPS'22)]

$$\theta^* = \operatorname{argmax}_{\theta \in P} \mathcal{L}(\theta) = \sum_i \log U_i(\theta)$$

Theorem 2. If U_i 's are concave, then θ^* is in the CORE. And can be computed in poly-time.

Proof sketch. (1) $\forall \theta \in P, \sum_i \frac{U_i(\theta)}{U_i(\theta^*)} \leq n$

(2) Then the claim implies θ^* in CORE.

Other settings (participatory budgeting, discrete, ...) [Fain-Goel-Munagala'16, Fain-Munagala-Shah'18]

CORE in FL: Distributed Protocol

[Chaudhury, Li, Kang, Li, M (NeurIPS'22)]

$$\theta^* = \operatorname{argmax}_{\theta \in P} \mathcal{L}(\theta) = \sum_i \log U_i(\theta)$$

Theorem 3. CoreFed: Distributed federated learning protocol to find CORE when U_i 's are concave.

Proof sketch.

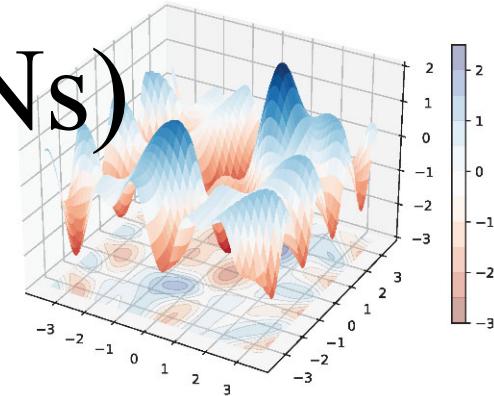
Solicit from agent i : $\partial_\theta \text{loss}_i(\cdot), \text{loss}_i(\theta)$.

Move in the direction of

$$\partial \mathcal{L}(\theta) = \sum_i \frac{\partial_\theta U_i(\cdot)}{U_i(\theta)} = \frac{\sum_i -\partial_\theta \text{loss}_i(\cdot)}{H - \text{loss}_i(\theta)}$$

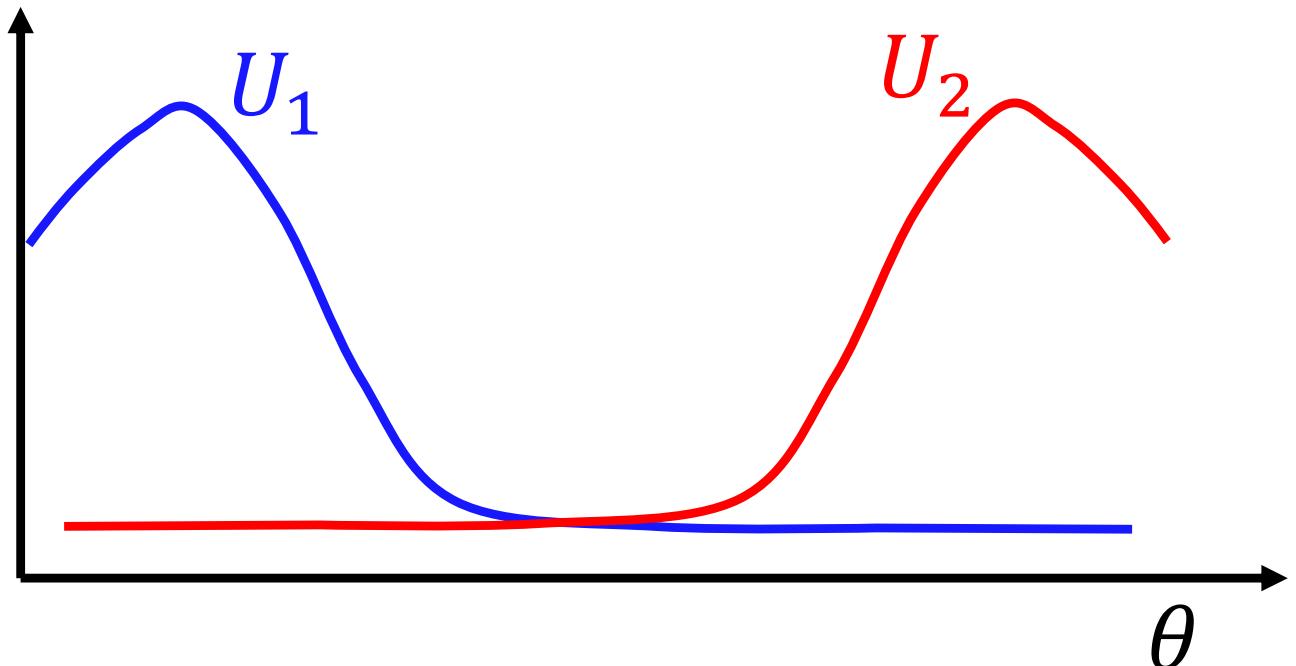
CORE in FL: Non-convex (DNNs)

[Chaudhury, Li, Kang, Li, M (NeurIPS'22)]



Local Guarantee: Local-approx. optima of $\mathcal{L}(\cdot)$ is in local-approx. pseudo CORE.

Anything Better?



DNNs: Experiments

[Chaudhury, Li, Kang, Li, M (NeurIPS'22)]

Setup: Two 5x5 convolution layers, 2x2max pooling, and two fully connected layer with ReLU activation.

Table 1: Comparison of utility ($M - \ell_{ce}$) for each agent trained with CoreFed and FedAvg. We see that $\sum_{i \in [n]} \frac{u_i(\theta')}{u_i(\theta^*)} < n$ holds, where θ' denotes the weights of shared model trained by FedAvg and θ^* by CoreFed.

| Dataset | Method | Agent 0 | Agent 1 | Agent 2 | U(Average) | U(Multi) | $\sum_{i \in [n]} \frac{u_i(\theta')}{u_i(\theta^*)}$ |
|----------|---------|---------|---------|---------|------------|----------|---|
| Adult | FedAvg | 2.59 | 0.77 | 1.46 | 1.61 | 2.91 | 2.80 (<3) |
| | CoreFed | 2.62 | 0.90 | 1.53 | 1.68 | 3.61 | |
| MNIST | FedAvg | 0.34 | 0.29 | 0.92 | 0.52 | 0.091 | 2.66 (<3) |
| | CoreFed | 0.36 | 0.41 | 0.91 | 0.56 | 0.13 | |
| CIFAR-10 | FedAvg | 0.63 | 1.40 | 0.51 | 0.84 | 0.45 | 2.62 (<3) |
| | CoreFed | 0.73 | 1.35 | 0.71 | 0.93 | 0.70 | |

CORE-style solution concept for DNNs?

Proportional Veto-CORE

[Chaudhury, Murhekar, Yuan, Li, **M**, Procaccia (ICML '24)]

(Ask me offline ☺)

Prop. Veto-CORE (Ordinal setting) [Moulin'81]

$$P = \{\theta_1, \dots, \theta_m\}$$

Agent i 's pref: $\theta_1^i \succ_i \theta_2^i \succ_i \dots \succ_i \theta_m^i$
($\equiv U_i$: $m \quad m-1 \quad \dots \quad 1$)

θ^* Proportional: $U_i(\theta^*) \geq \frac{\max_{\theta} U_i(\theta)}{n} = \frac{m}{n}.$

Agent i blocks θ^* if $U_i(\theta^*) < \frac{m}{n}$ ($B = \{\theta | \theta \succ_i \theta^*\}$)

Prop. Veto-CORE (Ordinal setting) [Moulin'81]

\mathbf{B}
 Agent i 's pref: $\theta_1^i \succ_i \theta_2^i \succ_i \dots \succ_i \theta^* \succ_i \dots \succ_i \theta_m^i$
 $(\equiv U_i: m \quad m-1 \quad \dots \quad 1 \quad)$

θ^* Proportional: $U_i(\theta^*) \geq \frac{\max_{\theta} U_i(\theta)}{n} = \frac{m}{n}.$

Agent i **blocks** θ^* if $U_i(\theta^*) < \frac{m}{n}$ ($B = \{\theta | \theta \succ_i \theta^*\}$)
 $\equiv (m - |B|) < \frac{m}{n} \equiv m \left(1 - \frac{|B|}{|P|}\right) < \frac{m}{n} \equiv \left(1 - \frac{|B|}{|P|}\right) < \frac{1}{n}$

Prop. Veto-CORE (Ordinal setting) [Moulin'81]

$$P = \{\theta_1, \dots, \theta_m\}$$

Agent i 's pref: $\theta_1^i \succ_i \theta_2^i \succ_i \dots \succ_i \theta^* \succ_i \dots \succ_i \theta_m^i$

Agent i blocks θ^* if $\left(1 - \frac{|B|}{|P|}\right) < \frac{1}{n}$ $(B = \{\theta | \theta \succ_i \theta^*\})$

Set $S \subseteq N$ blocks θ^* if $\left(1 - \frac{|B|}{|P|}\right) < \frac{|S|}{n}$
 $(B = \cap_{i \in S} \{\theta | \theta \succ_i \theta^*\})$

Veto-CORE: If no blocking coalition.

Prop Veto-CORE (Continuous setting)

[Chaudhury, Murhekar, Yuan, Li, M, Procaccia (ICML '24)]

P : Measurable set. λ : Measure function.

Agent i 's pref: $U_i: P \rightarrow \mathbb{R}_+$ measurable (allows DNNs)

$\theta^* \in P$. Set $S \subseteq N$ blocks θ^* if

$$\left(1 - \frac{\lambda(B)}{\lambda(P)}\right) \leq \frac{|S|}{n} \pm \epsilon$$

$\left(\exists B \subseteq P: \forall \theta \in B, \forall i \in S, U_i(\theta) \geq U_i(\theta^*) \right)$
atleast one strict

ϵ -Prop Veto-CORE: If no blocking coalition.

(Fair ML informs SCT!)

Prop Veto-CORE (PVC): Results

(**Continuous**) [Chaudhury, Murhekar, Yuan, Li, M, Procaccia (ICML'24)]

Theorem. If U_i 's are Lebesgue-measurable, then ϵ -Prop Veto-CORE exists for any $\epsilon \in (0, \frac{1}{n})$.

Proposition. If θ^* is in ϵ -PVC, then θ^* is

1. (approx.) Pareto-optimal
2. (approx.) (rankwise) Proportional

Proposition. Better guarantees for aligned preferences.

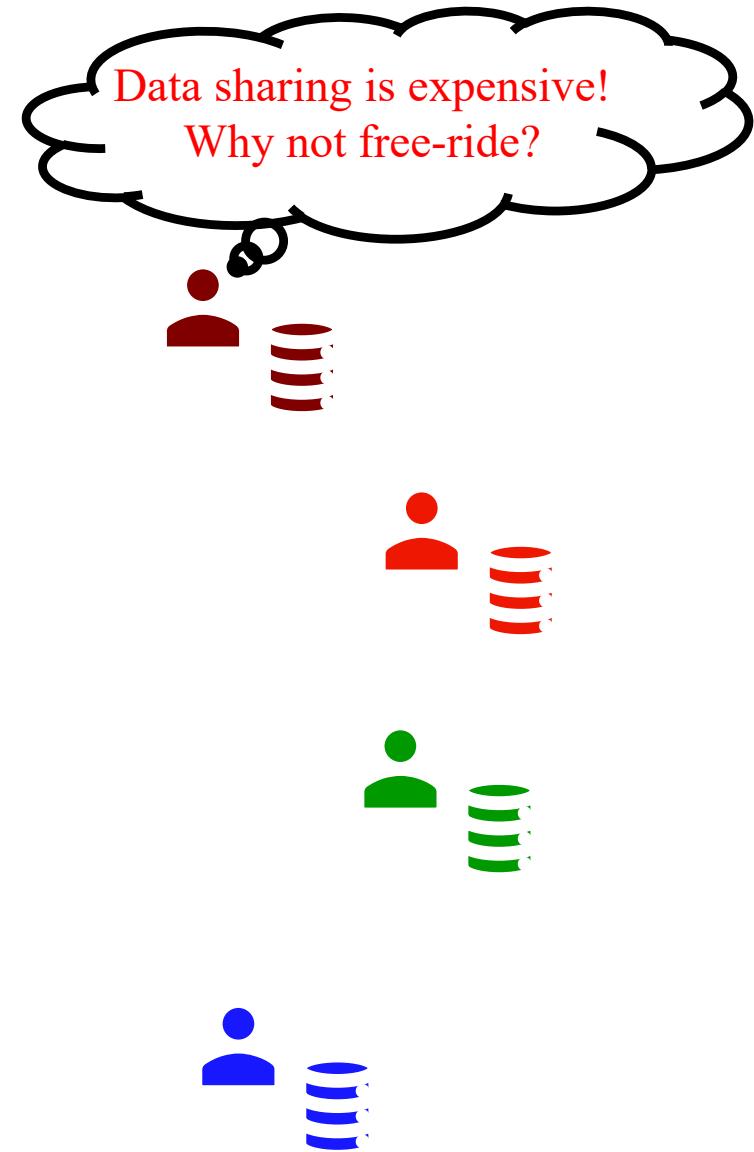
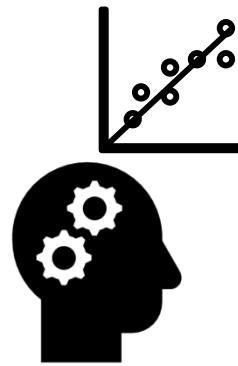
(Veto-)CORE: Questions

- Limited Heterogeneity: Better guarantees?
 - How to formalize heterogeneity parameter?
 - What guarantees are possible with respect to it?
- Strategic Analysis
 - Nash equilibrium, Truthful Mechanisms, ...



Data Sharing in FL: Incentives

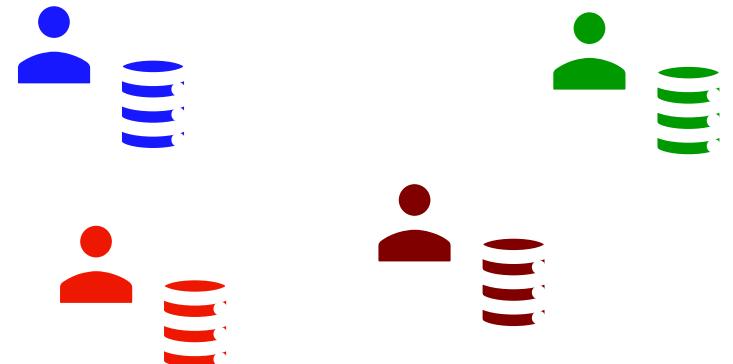
FL: Incentive Issues



Data sharing costs: computation/privacy/storage

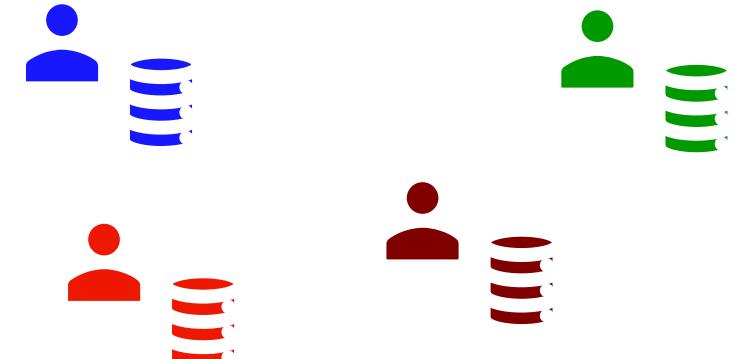
FL: Data Sharing Game

- A : set of n players/agents/clients
- Each agent $i \in A$ has
 - Has dataset D_i
 - **Strategy:** $d_i \in [0, 1]$ fraction of data shared
 - **Accuracy function** $a_i: [0, 1]^n \rightarrow R_+$
 - **Cost function** $c_i: [0, 1] \rightarrow R_+$ (cost of sharing data)



FL: Data Sharing Game

- N : set of n players/agents/clients
- Each agent $i \in A$ has
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 - Strategy: $d_i \in [0, 1]$ fraction of data shared
 - Accuracy function $a_i: [0, 1]^n \rightarrow R_+$
 - Cost function $c_i: [0, 1] \rightarrow R_+$ (cost of sharing data)
- **Nash Equilibrium (NE):** No unilateral deviation
For each agent i , $u_i(d_i, d_{-i}) \geq u_i(d'_i, d_{-i}), \forall d'_i \in [0, 1]$



Incentives: Prior Work

- [Blum, Haghtalab, Philips, Shao (ICML'21)]
 - Nash Eq. (NE) Analysis
 - Agent's goal: Minimize data shared subject to $a_i(\cdot) \geq \tau_i$
 - NE may not always exist. Sufficiency conditions, structural results.
- [Karimireddy, Guo, Jordan (Workshop@NeurIPS'22)]
 - Truthful mechanism to maximize data-sharing
 - Agent's goal: Maximize net payoff $u_i = a_i(d_1, \dots, d_n) - c_i(d_i)$
 - $c_i(d_i) = C_i * d_i$, and a_i 's are identical and concave
 - Grim-trigger style strategy

Welfare-maximizing? Fair? Budget-balanced?

Incentives in FL: Results

Agent's goal: Maximize net payoff $u_i = (\text{Accuracy} - \text{Cost})$

a_i 's concave, c_i 's convex

- [Murhekar, Yuan, Chaudhury, Li, **M** (NeurIPS'23)]
 - NE exists and can be reached via Best-Response-Dynamics.
 - NE may have bad welfare (due to free-riding)
 - Budget-balanced mechanism to maximize any p -mean welfare.
- [Murhekar, Song, Shahkar, Chaudhury, **M** (ICML'25)]
 - **Reciprocally fair mechanism, with payments p_i to agent i .**
 - Budget-balanced
 - Data + Accuracy gain



**Reciprocal Fairness:
(Karma!) You get what you give**

Reciprocity: You get what you give

Agent's goal: $u_i(\mathbf{d}) = a_i(\mathbf{d}) - c_i(d_i) + p_i$, where $\mathbf{d} = (d_1, \dots, d_n)$, p_i is payment.

$\phi_i^A(\mathbf{d})$ = Contribution of agent i to the welfare of other agents.

■ Shapley Value:

$$\phi_i^A(\mathbf{d}) = \sum_{S \subset A} \binom{n}{|S|}^{-1} (A(\mathbf{d}[S \cup \{i\}]) - A(\mathbf{d}[S]))$$

$$\mathbf{d}[S] = ((d_i)_{i \in S}, 0, \dots, 0) \text{ and } A(\mathbf{d}) = \sum_{i \in N} a_i(\mathbf{d})$$

Reciprocity of a Mechanism:

You get what you give

Agent's goal: $u_i(\mathbf{d}) = a_i(\mathbf{d}) - c_i(d_i) + p_i$, where $\mathbf{d} = (d_1, \dots, d_n)$, p_i is payment.

$\phi_i^A(\mathbf{d})$ = Contribution of agent i to the welfare of other agents.

M : Payment Mechanism, $NE(M)$: NE set of M

$$\text{Reciprocity}(M) = \min_{\mathbf{d} \in NE(M)} \min_{i \in A} \frac{a_i(\mathbf{d}) + p_i}{\phi_i^A(\mathbf{d})}$$

Claim. $\text{Reciprocity}(M) \leq 1$

Reciprocal Mechanism: M^{shap}

Shapley Value: $\phi_i^A(\mathbf{d}) = \sum_{S \subset A} \binom{n}{|S|}^{-1} (A(\mathbf{d}[S \cup \{i\}]) - A(\mathbf{d}[S]))$

■ M^{shap}

$$p_i(\mathbf{d}) = \phi_i^A(\mathbf{d}) - a_i(\mathbf{d})$$

Theorem(s). a_i concave, c_i convex non-decreasing, $\forall i$

- M^{shap} admits a NE, and Best Response converges quickly.
- $\text{Reciprocity}(M^{shap}) = 1$
- High Data-gain and Accuracy gain.

Incentives in FL: Results

Agent's goal: Maximize net payoff (Net utility – Cost)

U_i 's concave, c_i 's convex

- [Murhekar, Chaudhury, M'23]
 - NE exists and can be reached via Best Response Dynamics.
 - NE may have bad welfare (due to free-riding)
 - Budget-balanced mechanism to maximize any p -mean welfare.
linear, cross-entropy loss,
- [Murhekar, Chaudhury, M'24]
 - Reciprocally fair mechanism, with payments p_i to agent i .
Net utility $(a_i(\cdot) + p_i)$ of an agent is exactly equals her contribution to the collaboration aka her Shapley share
 - Budget-balanced

Open Directions

Incentives in FL: Data Sharing Game

- FL (distributed) protocols
- Non-IID data / Non-monotone accuracy
- Truthful Mechanisms
 - Without payment: fair / welfare-maximizing
 - With payments: budget-balanced / fair/ welfare-maximizing
- (Data) Contracts

General Direction:

Fair/Trustworthy ML via GT+SCT



Mintong Kang



Aniket Murhekar



Zhuowen Yuan



Jiaxin Song



Bhaskar R. Chaudhury



Bo Li



Linyi Li



Ariel Procaccia

THANK YOU